E872 Note
Using Individual Event Analysis to Calculate Relative Probabilities for Tau and Charm Candidates

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1 Introduction

During the last collaboration meeting, George requested that we each write a note to summarize the method and results of our analysis. This is my attempt to explain and summarize the results of the parameter analysis I have performed on the τ and charm candidates.

2 Event Parameters

2.1 Kink Events

In order to describe these parameters, I must first define terms. The parent track is the track that decays or interacts. This could be a τ, a charm particle, or a hadron in this analysis. For the kink events the parent decays or interacts to produce a single charged particle, which is the daughter.

The five parameters used in this analysis are θ, the angle between the neutrino direction and the parent track direction, $\Delta\phi$, which is the asymmetry of the produced tracks' ϕ angles, the decay length, L, which is the length of the parent track, the daughter momentum, P, and α, the angle between the parent and the daughter track. The value of θ is obtained by measuring the z distance of the parent track and the projection of the parent track in the plane perpendicular to the beam direction. With these two sides, the angle is calculated. The value of $\Delta\phi$ is obtained by measuring the polar angle for each track. The angle between the parent track and the net ϕ angle of the other tracks is calculated. The net of the polar angles of the other tracks is calculated by treating each track as a unit vector and adding them. The decay length, L, is measured from the data. The momentum of the daughter, P is measured using the spectrometer and multiple scattering in the emulsion. The angle between the daughter and the parent track is measured using a similar method as the one used to measure θ. The values of the parameters for each kink event are listed in Table 1 for each kink candidate. The material in which the decay or interaction occurred in is also listed in this table.

2.2 Trident Events

The four parameters used in this analysis are θ, $\Delta\phi$, the decay length, L, and the sum of the impact parameters of the daughters, ΣIP. The first three parameters

Event	θ (rad)	$\Delta\phi$ (rad)	L (mm)	α (rad)	P (GeV)	Material
3024_30175	0.028	1.09	4.59	0.093	2.9	Plastic
3263_25102	0.169	0.10	1.97	0.130	1.9	Iron
3039_01910	0.067	2.71	0.29	0.090	4.6	Plastic
3333_17665	0.016	2.84	0.55	0.013	21	Plastic
3065_03238	0.250	0.71	2.10	0.229	1.2	Iron
3193_01361	0.088	0.46	1.86	0.019	17	Iron
3073_22977	0.031	1.36	1.00	0.065	4.2	Emulsion
2986_00355	0.065	3.07	6.52	0.008	1.3	Emulsion

Table 1: The τ and charm kink candidates and their values for the 5 parameters used for this analysis.

are explained in the previous section. The IP is calculated by multiplying L, the length of the parent track and the sine of the angle between the parent track and the daughter track. This is calculated for all three daughter particles and added to calculate ΣIP. The parameters for each of the trident events are listed in Table 2.

Event	θ (rad)	$\Delta\phi$ (rad)	L (mm)	Σ IP (mm)	Material
3334_19920	0.040	3.11	8.87	0.0363	Plastic
3296_18816	0.141	1.74	0.78	0.0293	Emulsion
3245_22786	0.142	16	0.40	0.0128	Emulsion

Table 2: The τ and charm trident candidates and their values for the 4 parameters used for this analysis

3 Overview of Analysis

The relative probability that an event occurs is calculated using Bayes's equation, Eqn. (1).

$$P(i) = \frac{PP_i \Pi(i)}{PP_i \Pi(i) + \Sigma_{bkg} PP_{bkg} \Pi(bkg)} \tag{1}$$

where i is the three different hypothesizes considered. The three hypotheses considered are: a nu_τ interaction which produces a τ decay, a ν_μ or ν_e interaction which produces a charm decay or hadron interaction. The decays or interaction must result in three charged particles. The background term, $\Sigma_{bkg} PP_{bkg} \Pi(bkg)$, is the sum over all of the possibilities except the ith possibility. PP_i is the prior probability of the event being an i type event. $\Pi(i)$ is the probability density function evaluated at a set of measured values of event type i.

PP_i is the priori knowledge of the likelihood of event type i.This is calculated using branching ratios, production cross sections, and efficiencies.

$\Pi(i)$ is calculated using simulated data for each hypothesis, and his is measured by calculating the fraction of simulated events which reside in a region of parameter space centered on the specific values of the parameters for each event.

4 Prior Probabilities

The prior probability is the expected number of a certain type of event divided by the total number of events. Since the total number events is constant equal to 417 events, the expected number is used for this analysis.

4.1 Prior Probabilities for Trident Events

Each event has specific characteristics which are used in this analysis. The more characteristics included in the analysis, the more accurate the analysis is. The prior probabilities include as much as information as possible, therefore, there are many different prior probabilities calculated. Two tau candidates, 3334_19920 and 3296_18816, and one charm candidate, 3245_22786, make up the set of trident events. The charm trident event has identified muons from both the primary and secondary vertex; therefore, the prior probability is calculated for this specific case for the charm candidate. In this section prior probabilities are calculated for the following event types: τ trident decay without an identified lepton from the primary, τ trident decay with identified muons from both the primary and secondary vertexes, charm trident decay without an identified lepton from the primary, charm trident decay with identified muons from both the primary and secondary vertexes, hadronic interaction which results in three charged particles without an identified lepton from the primary, and hadronic interaction which results in three charged particles with identified muons from both the primary and the secondary vertexes.

4.1.1 Prior Probability of Tau Decay with No Identified Leptons

The expected number of τ trident events, $N_{\tau\text{trid}}$, is calculated using equation (2).

$$\langle N_{\tau\ \text{trid}}\rangle = N_\tau BR(\tau \rightarrow \text{trident}) \tag{2}$$

where N_τ is the number of expected ν_τ interactions in the data and $BR(\tau \rightarrow \text{trident})$ is the branching ratio for a τ to decay to three charged particles. $BR(\tau \rightarrow \text{trident}) = 0.1457$ [1]. N_τ is the number of ν_τ interactions measured from the number of ν_e and ν_μ interactions found in the data set, the predicted rates of the each neutrino flavor in the beam, and the total efficiency in locating each kind of neutrino interaction, which comes from simulated data. The total number of ν_τ interactions expected in the 203 data set is 8.2 events, or 4.0 % of the neutrino interactions are expected to be ν_τ interactions [2]. The expected number of ν_τ interactions in the 417 data set is 16.7 neutrino events and $\langle N_{\tau\text{trid}}\rangle$ is 2.40 events.

4.1.2 Prior Probability of Tau Decay with Identified Muons from the Primary and Secondary Vertexes

If there is an identified μ from the primary vertex, the prior probability of a τ event is 0.0.

4.1.3 Prior Probability of Charm Decay with No Identified Lepton

The expected number of background charm events is given by equation (3).

$$\langle N_{\text{c trid}}\rangle = \sum_i N_{\text{c}_i} BR(\text{C}_i \rightarrow \text{3-prong}) P_{\text{sel}}, \tag{3}$$

where N_{c_i} is the expected number of charm particles of type i in the data set, $BR(\text{c}_i \rightarrow \text{3-prong})$ is the probability that a charm particle of type i will decay to produce three charged particles, and P_{sel} is the probability that the charm event will pass the τ selection criteria. The expected number of charm particles of type i is calculated using equation (4).

$$N_{\text{c}_i} = N_{cc} F \alpha_i, \tag{4}$$

where N_{cc} is the number of charged current events, F is the average charm particle production fraction, and α_i is the fraction of time that the charm quark will produce a charm particle of type i, where i is D, D_s, or Λ_c. In the 203 data set, the number of charged current interactions is 154 [2]. If the ratio of charged current events:neutral current events is constant for the 417 data set, then the 417 data set should contain 316 charged current events. Currently 295 events have been identified as charged current events out of the 417 data set. For E872's energies, the fraction F is 0.066 [2]. This result comes from the results of E531, CCFR μ^-, and CCFR μ^+ data. Table 3 summarizes α_i for each charm particle: [3]

Particle	α
D	0.24
D_s	0.09
Λ_c	0.10

Table 3: The α values for each charm particle

The expected number produced for each type of charm particle is listed in Table 4 with $BR(\text{c}_i \rightarrow \text{3-prong})$ [1], P_{sel}, and the expected number of background charm events.

Particle	N_{c_i}	$BR(c_i \to \text{3-prong})$	P_{sel}	$\langle N_{\text{c trid}} \rangle$
D	5.00	0.37	0.48	0.88
D_s	1.88	0.32	0.48	0.28
Λ_c	2.09	0.20	0.48	0.20
Total				1.36

Table 4: Summary of values required to calculate charm prior probabilities for trident events with no identified leptons

4.1.4 Prior Probability of Charm Decay with Identified Muons from the Primary and Secondary Vertexes

The prior probability for this charm event changes slightly if there is an identified μ from the primary and the secondary. Eqn 3 becomes:

$$\langle N_{\text{c trid}} \rangle = \sum_i N_{c_i} BR(c_i \to \mu\text{-trid}) \tag{5}$$

where P_{sel} is no longer present the charm candidate does not have to pass all of the tau cuts, and $BR(c_i \to \mu\text{-trid})$ is the probability of the charm decaying to three charged particles, one of which is a μ. There are two scenarios which result in a μ in the secondary: the charm particle decays to two charged particles and a μ or the charm particle decays to three charged particles, one of them a π, and the π subsequently decays to a μ.

For the first scenario, all of the branching ratios of charm to μ for the relevant charm particles must be summed. The branching ratios I used are summarized in Table 5.

Parent	Decay Mode	BR
$D^{\pm}$	$K^-\pi^+\mu^-\nu_\mu$	0.032
$D^{\pm}$	$\rho^0\mu^+\nu_\mu$	0.0027
$D^{\pm}$	Total	0.0347
D_s	$\phi l^+\nu_l$	0.02
D_s	Total	0.02

Table 5: Branching ratios of charm decays to μ-tridents

The expected number of charm and sums of the branching ratios are summarized in Table 6

The second scenario requires a slightly different expression for the expected number:

$$\langle N_{\text{c trid}} \rangle = \sum_i N_{c_i} BR(c_i \to \pi\text{-trid}) P(\pi \to \mu\nu_\mu) \tag{6}$$

Type	N_{c_i}	$BR(\mu\text{-trid})$	$\langle N_{\mathrm{c\ trid}}\rangle$
D	5.00	0.033	0.165
D_s	1.88	0.020	0.038
Λ_c	2.09	0.000	0.000
Total			0.203

Table 6: Expected number for charm trident decays with an μ from both the primary and the secondary vertexes. In this case, the μ is a direct decay product

where $BR(\mathrm{c}_i \rightarrow \pi\text{-trid})$ is the sum of the branching ratios for the i type charm particle to decay to three charged particles including at least one π. $P(\pi \rightarrow \mu\nu_\mu)$ is the probability that the π will decay to a μ before the μ-ID, which is 11 m from target. The probability of having a μ from the secondary vertex is related to the probability that the pion will decay before the μ-ID. $P(\pi \rightarrow \mu\nu_\mu) = \ 1 - e^{\frac{-t}{\tau}}$ where: $\tau = 2.6\text{x}10^{-8}$, $P_\pi = 10GeV \simeq E$ and $t = \frac{L}{\gamma} = \frac{L}{\frac{E}{m}} = 0.51ns$ Using these values $P(\pi \rightarrow \mu\nu_\mu) = 1 - e^{-t/\tau} = 0.0195$.

The branching ratios for charm decays to three charged particles including at least one π are summarized in tables 7, 8, and 9. The results for the expected number is shown in Table 10.

Parent	Decay Mode	BR	N_π	BRN_π
Λ_c	$pK^-\pi^+$	0.005	1	0.005
Λ_c	$p\bar{K}^0\pi^+\pi^-$	0.026	2	0.052
Λ_c	$pK^-\pi^+\pi^0$	0.034	1	0.034
Λ_c	$pK^-\pi^+\pi^0\pi^0$	0.008	1	0.008
Λ_c	$p\pi^+\pi^-$	0.0035	2	0.007
Λ_c	$\Lambda\pi^+\pi^+\pi^-$	0.033	3	0.099
Λ_c	$\Sigma^+\pi^+\pi^-$	0.036	2	0.072
Λ_c	$\Sigma^-\pi^+\pi^+$	0.019	2	0.038
Λ_c	$\Sigma^0\pi^+\pi^+\pi^-$	0.011	3	0.033
Λ_c	$\xi^-K^+\pi^+$	0.0049	1	0.0049
Λ_c	$\Sigma^+K^+\pi^-$	0.0017	1	0.0017
Λ_c	Total			0.3546

Table 7: Branching ratios for Λ_c decays to a trident including a π

Parent	Decay Mode	BR	N_π	BRN_π
D_s^+	$K^+K^-\pi^+$	0.044	1	0.044
D_s^+	$K^+K^0\pi^+\pi^-$	0.025	2	0.05
D_s^+	$K^0K^-\pi^+\pi^+$	0.043	2	0.086
D_s^+	$\pi^+\pi^+\pi^-$	0.01	3	0.03
D_s^+	$K^+\pi^+\pi^-$	0.01	2	0.02
D_s^+	Total			0.230

Table 8: Branching ratios for D_s^+ decays to a trident including a π

Parent	Decay Mode	BR	N_π	BR*N_π
D^+	$K^-\pi^+e^+\nu_e$	0.041	1	0.041
D^+	$K^-\pi^+\pi^+$	0.091	2	0.182
D^+	$K^-\pi^+\pi^+\pi^0$	0.064	2	0.128
D^+	$\bar{K}^0\pi^+\pi^+\pi^-$	0.07	3	0.21
D^+	$K^+K^-\bar{K}^0\pi^+$	0.00054	1	0.00054
D^+	$\bar{K}^0\rho^0\pi^+$	0.042	2	0.084
D^+	$\bar{K}^0\pi^+\pi^+\pi^-\pi^0$	0.054	3	0.162
D^+	$K^*\pi^+\pi^+\pi^-$	0.0082	3	0.0246
D^+	$\pi^+\pi^+\pi^-$	0.0031	3	0.0093
D^+	$\pi^+\pi^+\pi^-\pi^0$	0.00069	3	0.00207
D^+	$\rho^0\pi^+$	0.001	1	0.001
D^+	$K^+K^-\pi^+$	0.0088	1	0.0088
D^+	$K^+K^-\pi^+\pi^0$	0.026	1	0.026
D^+	$K^+\bar{K}^0\pi^+\pi^-$	0.004	2	0.008
D^+	$K^0K^-\pi^+\pi^+$	0.0054	2	0.0108
D^+	$\phi\pi^+$	0.0061	1	0.0061
D^+	$\phi\pi^+\pi^0$	0.023	1	0.023
D^+	Total			0.9272

Table 9: Branching ratios for D^+ decays to a trident including a π

The total expected number of charm events with an identified μs from the primary and the secondary is 0.31.

4.1.5 Prior Probability of Hadronic Interaction with No Identified Leptons

The expected number of hadronic interaction background events is calculated using equation (7).

$$\langle N_{\text{had trid}} \rangle = L_i P(\text{int})_i P_{\text{sel}}, \tag{7}$$

Type	N_{c_i}	$BR * N_\pi$	$P(\pi \to \mu\nu_\mu)$	$\langle N_{\text{c trid}} \rangle$
D	5.00	0.927	0.0195	0.0903
D_s	1.88	0.230	0.0195	0.0084
Λ_c	2.09	0.355	0.0195	0.0132
Total				0.1058

Table 10: The expected number for charm trident events with identified μ from both the primary and secondary vertexes. In this case the μ results from a decay of a π, which originates in the charm decay.

where L_i is the total length of all the hadron tracks through material i, and i is the material in which the decay or interaction occurred. i is either iron, emulsion, or plastic. $P(\text{int})_i$ is the probability that the hadrons will interact to produce three charged particles in material i per unit length. This is calculated using the GEANT detector simulation. P_{sel} is the probability that the hadronic event passed that τ selection criteria. These values are summarized in Table 11.

Material	$P(\text{int})$	L_i	P_{sel}	$\langle N_{\text{had trid}} \rangle$
Iron	3500	5.88 x 10^{-4}	0.48	1.0
Emulation	2700	3.64 x 10^{-4}	0.48	0.47
Plastic	1800	2.07 x 10^{-4}	0.48	0.18

Table 11: Prior Probabilities for trident hadronic interaction with no identified leptons

4.1.6 Prior Probability of Hadronic Interaction with Identified Muons from the Primary and Secondary Vertexes

Eqn. 7 must be slightly modified for this prior probability. The following expression is used to calculate this prior probability:

$$\langle N_{\text{had trid}} \rangle = L_i P(\text{int}) P(\pi\text{-trid}) P(\pi \to \mu\nu_\mu) \tag{8}$$

where L_i is the total length of material i traversed, and i is the material in which the decay or interaction decayed, $P(\text{int})$ is the probability that a hadron interacts in material i and produces three charged particles, $P(\pi\text{-trid})$ is the probability that one of the charged particles is a π, and $P(\pi \to \mu\nu_\mu)$ is the probability that the π will decay to a μ before the μ-ID. I used the MC to calculate $P(\pi\text{-trid})$, which is 99%. The results of this calculation are summarized in Table 12.

4.1.7 Results

The expected number of each type of event are summarized in Table 14.

Material	$P(\text{int})$	L_i	$P(\pi \to \mu\nu_\mu)$	$\langle N_{\text{had trid}} \rangle$
Iron	3500	5.88 x 10^{-4}	0.0195	0.04
Emulsion	2700	3.64 x 10^{-4}	0.0195	0.02
Plastic	1800	2.07 x 10^{-4}	0.0195	0.01

Table 12: Summary of prior probability for trident hadronic interactions with identified muons from the primary and secondary vertexes

Event	$\langle N_{\text{tau}} \rangle$	$\langle N_{\text{charm}} \rangle$	$\langle N_{\text{hadron}} \rangle$
3334_19920	2.40	1.36	0.18
3296_18816	2.40	1.36	0.47
3245_22786	0.00	0.31	0.02

Table 13: This table summarizes the expected numbers for each of trident event for each hypothesis

4.2 Prior Probabilities for Kink Events

A detailed analysis of the tau kink candidate was performed by Jason, and is the subject of his thesis. I will, therefore, simply quote his results and refer to his thesis [2] for a more detailed explanation. For the charm kink candidates, the expected numbers for the following event types are calculated: τ kink decay with identified lepton from the primary, charm kink decay with identified lepton from the primary, and hadronic interaction which results in one charged track with an identified lepton from the primary.

4.2.1 Prior Probability of Tau Decay with an Identified Lepton from the Primary Vertex

If there are identified leptons from the primary, the prior probability of τ is 0.00.

4.2.2 Prior Probability of Charm Decay with an Identified Lepton from the Primary Vertex

The prior probability for a charm decay is calculated using:

$$\langle N_{\text{c kink}} \rangle = \sum_i N_{\text{c}_i} BR(\text{c}_i \to \text{1-prong}) P_{\text{sel}} \tag{9}$$

where these terms are defined in Section 4.1.3. The expected numbers of charm events are listed in table 16, along with the branching ratios and P_{sel}[2].

Particle	N_{c_i}	$P(c_i \to 1\text{-prong})$	P_{sel}	$\langle N_{c\ kink} \rangle$
D	5.00	0.46	0.0.132	0.30
D_s	1.88	0.37	0.0.138	0.10
Λ_c	2.09	0.65	0.096	0.13
Total				0.53

Table 14: Expected number for charm kink candidates with no identified lepton from the primary vertex

4.2.3 Prior Probability of Hadronic Interaction with Identified Leptons from the Primary Vertex

These values also come from Jason's analysis. Each possible type of background hadronic interaction which occurs in a significant number is listed in Table 15 along with the probability of interacting per mm, $P(\text{int})$, L_i, P_{sel}, and the expected number of interaction.

Material	L_i	$P(\text{int})$	P_{sel}	$\langle N_{had\ kink} \rangle$
Iron	3500	1.28 x 10^{-4}	0.48	0.24
Emulsion	2700	1.27 x 10^{-5}	0.48	0.03
Plastic	1800	3.5 x 10^{-5}	0.48	0.02

Table 15: Expected number of hadronic kink interactions with no identified leptons from the primary vertex

4.3 Results

The expected numbers for each kink event is listed in Table 16.

Event	Tau	Charm	Hadron
3024_30175	0.854	0.044	0.0
3333_17665	0.854	0.028	0.0
3039_01910	2.2	0.19	0.015
3263_25102	2.2	0.17	0.10
2986_00355	0.00	0.53	0.06
3073_22977	0.00	0.53	0.06
3193_01361	0.00	0.53	0.29
3065_03238	0.00	0.53	0.29

Table 16: Expected Number for the single prong τ and charm candidates

5 Probability Densities

A list of simulated events were generated using the LEPTO event generator. This list gives the weight and the values for all of the necessary parameters. A volume in parameter space is defined. This is defined by choosing a small interval in each parameter which is centered on the event's value of this parameter. The kink events used a 5-D parameter volume while the trident events use a 4-D parameter volume. $\Pi(i)$ is defined as:

$$\Pi(i) = \frac{N_{\Delta V}}{N_T \Delta V} \tag{10}$$

where ΔV is the volumed defined in parameter space, $N_{\Delta V}$ is the weight of simulated events that reside in the ΔV, and N_T is the total weight of all the simulated events. The results are summarized in Table 19.

Event	PDF(τ)	PDF(charm)	PDF(hadron)
3024_30175	0.17	2.07	—
3263_25102	0.01	0.27	0.98
3039_01910	16.7	7.80	0.26
3333_17665	14.1	2.42	—
3356_17099	0.00	1.13	0.13
3065_03238	0.32	2.02	0.45
3193_01361	4.30	6.46	0.30
3073_22977	44.1	8.73	0.00
2986_00355	97.8	426	0.11
3334_19920	403.5	1.22	2.48
3296_18816	22.2	12.8	7.40
3245_22786	32.2	222	210

Table 17: Probability Densities for all events

6 Results

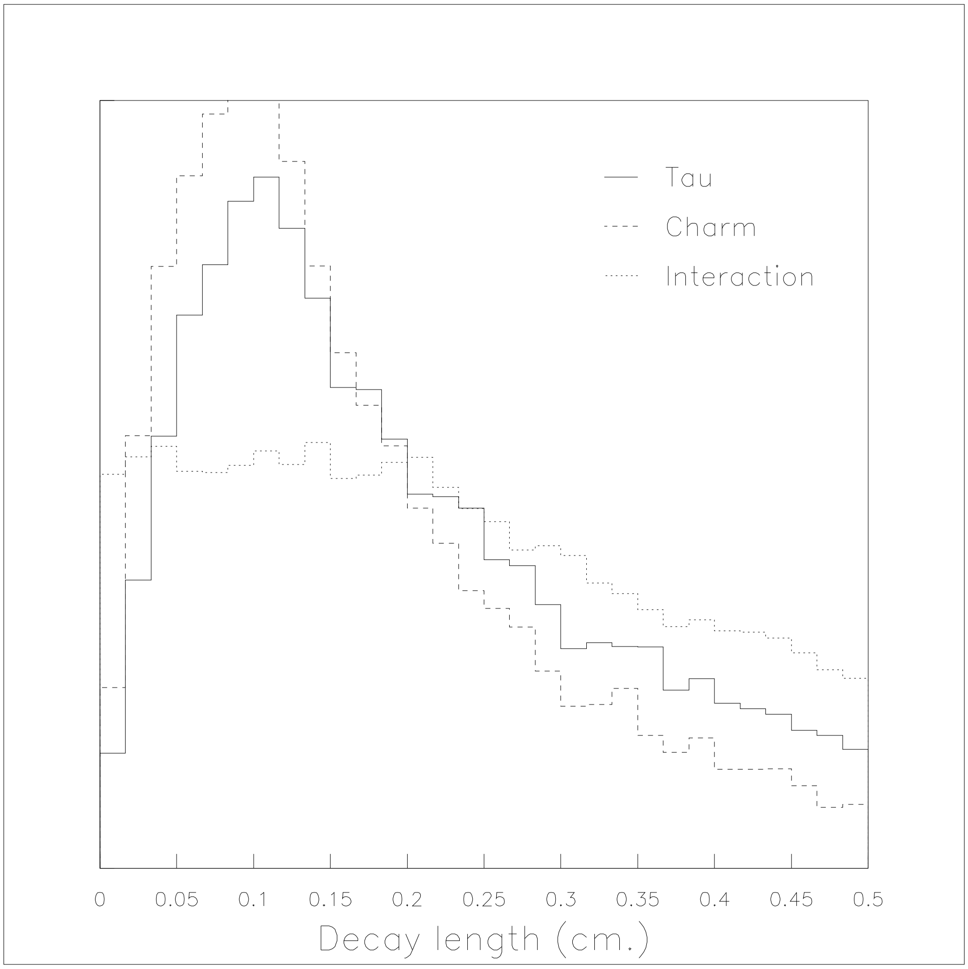
The probabilities were calculated for each type using (1), Table 16, and Table 19 and are listed in Table 18. The last column simply lists which hypothesis has the highest probability.

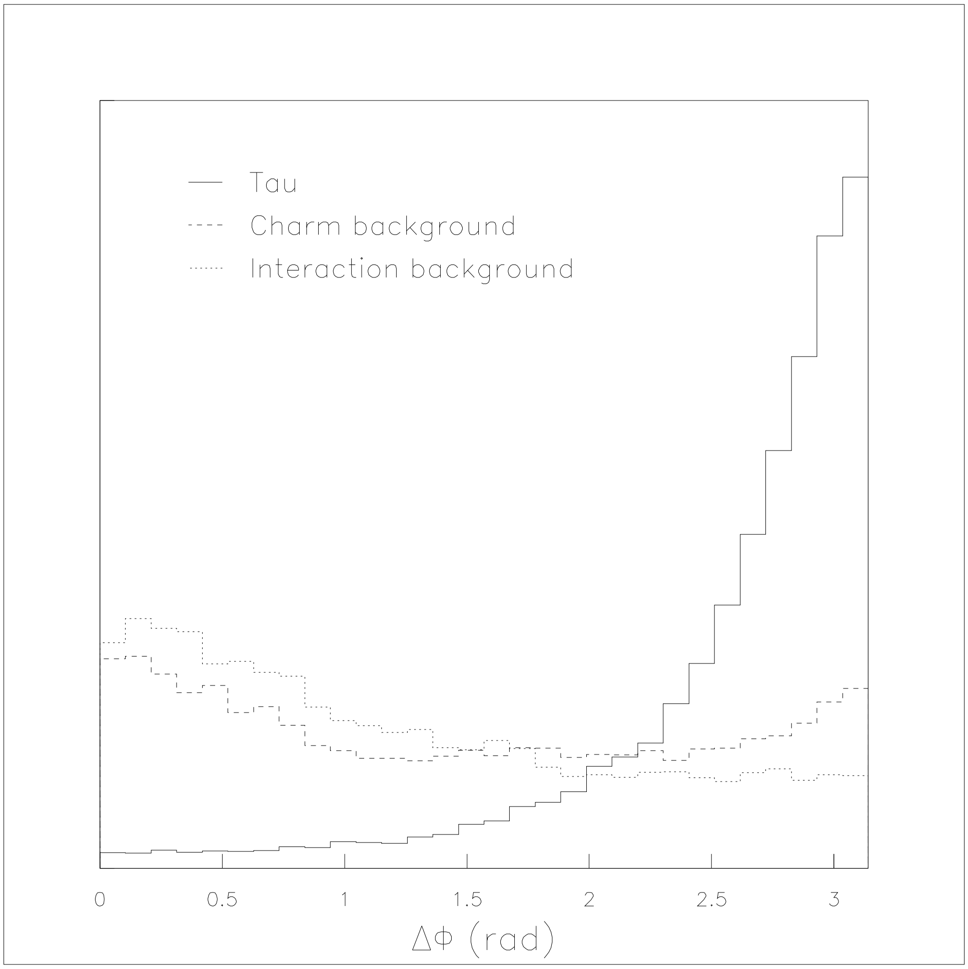
Event	P(τ)	P(charm)	P(hadron)	Type	Material
3024_30175	0.67	0.33	0.00	Tau	Plastic
3039_01910	0.98	0.02	3 x 10^{-4}	Tau	Plastic
3263_25102	0.14	0.27	0.59	Hadronic	Iron
3296_18816	0.72	0.23	0.05	Tau	Emulsion
3333_17665	0.98	0.02	0.00	Tau	Plastic
3334_19920	0.999	0.002	0.003	Tau	Plastic
2986_00355	0.00	0.999	0.001	Charm	Emulsion
3065_03238	0.00	0.89	0.11	Charm	Iron
3073_22977	0.00	0.99	0.01	Charm	Emulsion
3193_01361	0.00	0.97	0.03	Charm	Iron
3245_22786	0.00	0.94	0.06	Charm	Emulsion

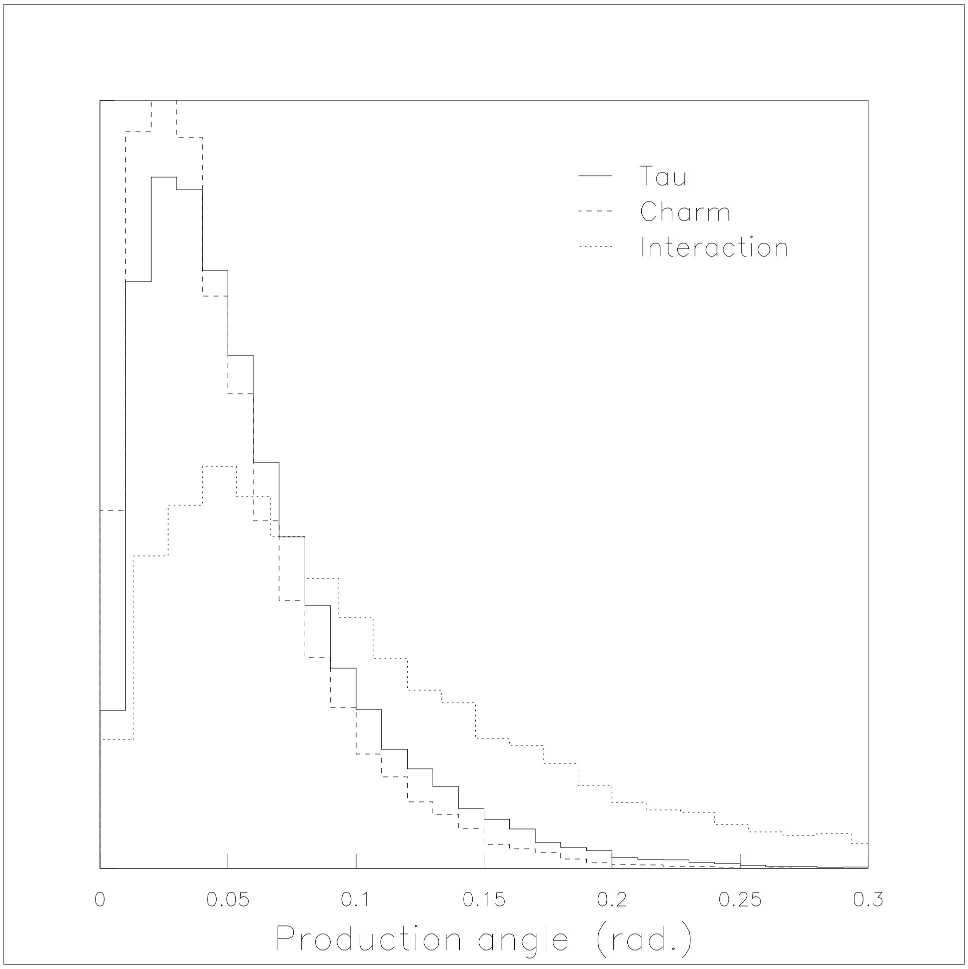
Table 18: Relative Probabilities for all events where P(τ) is the relative probability that the event is a ν_τ events, P(charm) is the relative probability that the event is a charm background event, and P(hadron) is the relative probability that the event is a hadronic interaction background event

7 Distributions

The following are the Monte Carlo distributions I used to calculate the probability density function.




Tau
Charm background
Interaction background
0
0.5
1
1.5
2
2.5
3
ΔΦ (rad)


Tau
Charm
Interaction
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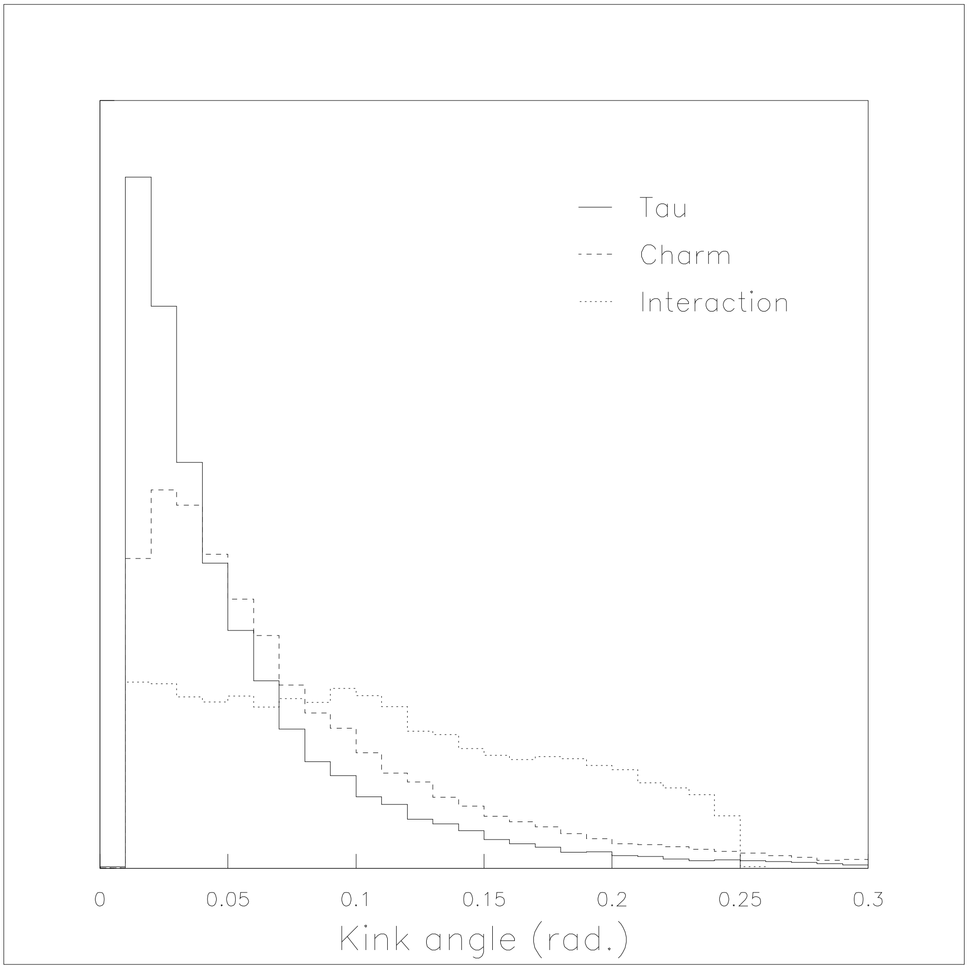
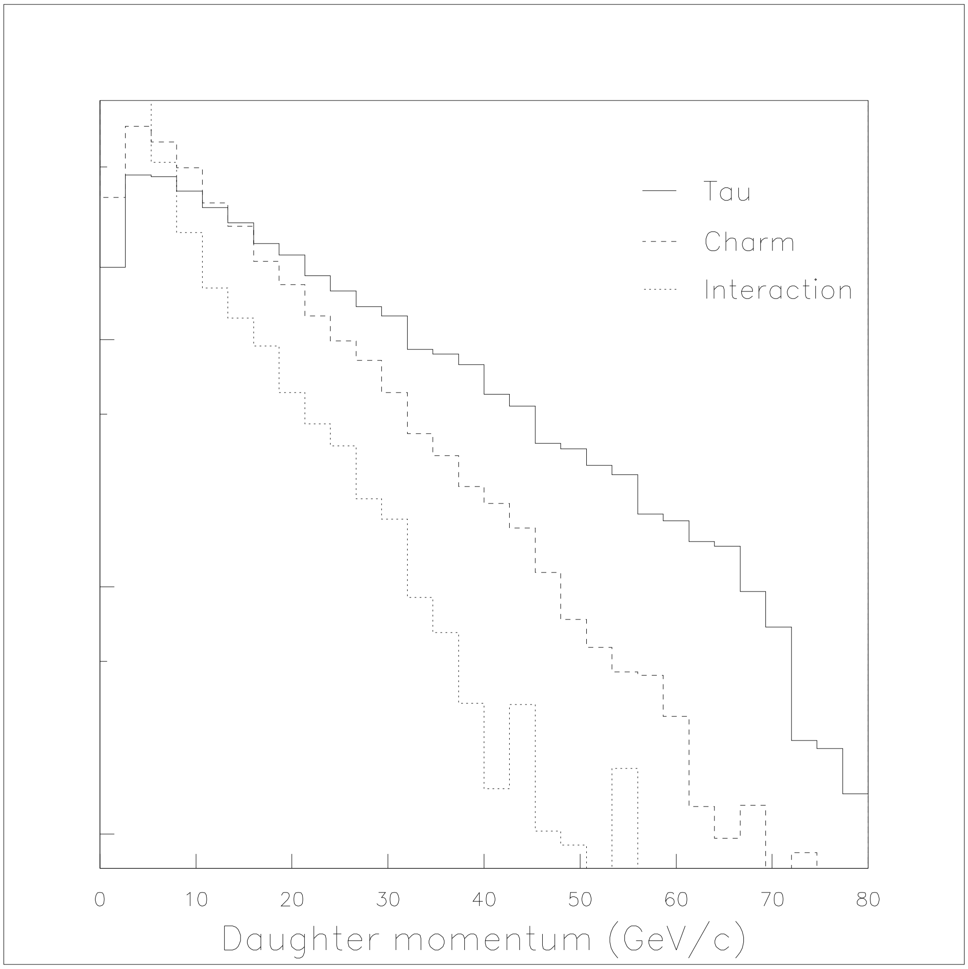
0.05
0.1
0.15
0.2
0.25
0.3
Production angle (rad.)


Tau
Charm
Interaction
0
0.05
0.1
0.15
0.2
0.25
0.3
Kink angle (rad.)



References

[1] Particle Data Group: (2002)

[2] Jason Sielaff: Ph.D. Thesis (2002)

[3] T. Bolton: hep-ex/9708014v1 (1997)